微積分期末考 (2003 Spring) 系級: 學號: 姓名:

1. (a) Find the derivative of $f(x,y,z) = x^3 - xy^2 - z$ at $P(1,1,0)$ in the direction of $\vec{v} = 2\vec{i} - 3\vec{j} + 6\vec{k}$. (5分)
(b) In what direction does f decrease most rapidly at $P(1,1,0)$ and what is the rate of change in the direction. (5分)

2. Show that $\lim_{(x,y)\to(0,0)} \frac{x+y}{x-y}$ does not exist. (10分)

3. Find the parametric equation for the line tangent to the curve of intersection of the following two surfaces at the point $(1,0,3)$. (20分)

$$x^2 + y^2 = 1, \quad x + z = 4.$$

4. Find the absolute maxima and minima of the function $f(x,y) = (4x - x^2)\cos y$ on the rectangular plate $1 \le x \le 3$, $-\pi/4 \le y \le \pi/4$. (10分)

5. Use the Method of Lagrange Multiplies to find the maximum and minimum values of the function $f(x,y) = 3x+4y$ on the circle $(x-1)^2+y^2 = 1$. (20分)

6. Suppose $\int\int_R f(x,y)dA = \int_0^2 \int_{x^2}^{2x} f(x,y)dydx$. Please sketch the region R. (10分)

7. Find the area enclosed by one leaf of the rose $r = \cos 3\theta$. (10分)

8. Find the volume of the D cut from the solid sphere $\rho \leq 1$ by the cone $\phi = \pi/3$ in the spherical coordinates. (10分)